Transiting extrasolar planets: Calculating some important data

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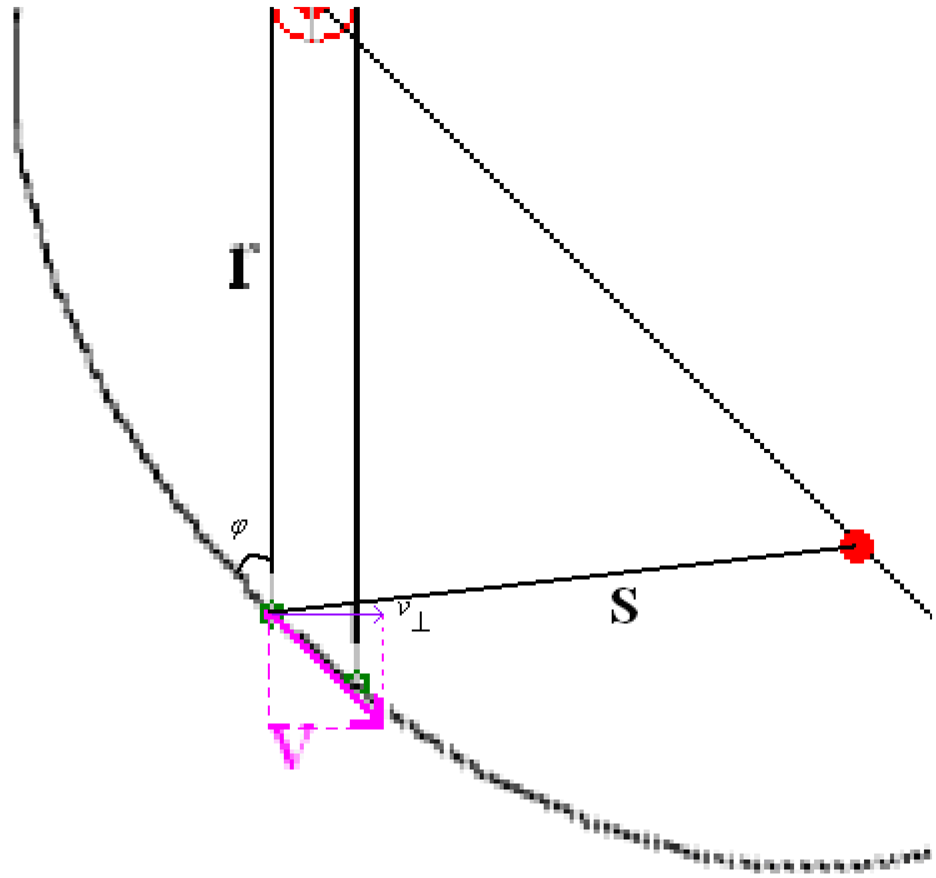
Approximations:

- stars oblateness negligible
- orbtital velocity constant during the transit
- Orbital curvature negligible during the transit
- 2 bodies orbital system

Central transit duration (D_{Max} ➔ i = 90°):

Usually the transit duration is different from the maximum duration, which occurs only when the inclination to the observers is exactly 90°. The maximum transit duration is important to discover some useful data, like the inclination itself



$$D_{Max} = \frac{2R_{Tot}}{v_{Tot\perp}}$$

The component of the total velocity perpendicular to r is:

$v_{Tot\perp} = v_{Tot} \sin\varphi$

$$\sin\varphi = \sqrt{\frac{(r_P + s_P)^2 - d_P{}^2}{4 r_P s_P}}$$

Then: $D_{Max} = \dfrac{2R_{Tot}}{v_{Tot}\sin\varphi}$

Let's calculate v_{Tot} and $\sin\varphi$:

1) v_{Tot}

In order to estimate the transit maximum duration, we need to calculate the total velocity, which is the combined planet and star orbital velocity. Let's do that considering the two orbits and their angular velocities, that must be the same:

$\omega_S = \omega_P$ ➔ $\dfrac{v_P}{r_P} = \dfrac{v_S}{r_S}$ ➔ $v_S = \dfrac{r_S}{r_P} v_P$. Using the center of mass relation: $m_P r_P = m_S r_S$ we have:

$v_S = \dfrac{m_P}{m_S} v_P$ and the total velocity is: $v_{Tot} = v_P\left(1 + \dfrac{m_P}{m_S}\right)$

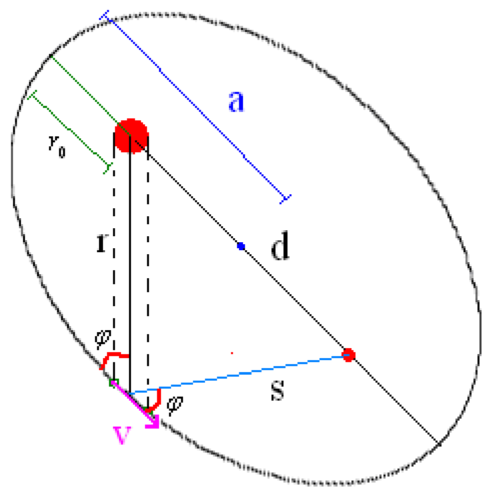
Using the orbital velocity relation for elliptical orbits: $v_P = \sqrt{GM\left(\dfrac{2}{r_P} - \dfrac{1}{a_P}\right)}$ and eliminating r_P and a_P using the definition of an ellipse: $r_P = \dfrac{a_P(1-e^2)}{1+e\cos\phi}$ and the third Kepler's law $a_P = \left(\dfrac{GP^2M}{4\pi^2}\right)^{1/3}$:

$$v_P = \sqrt{\left(\frac{2\pi GM}{P}\right)^{2/3}\left(\frac{2(1+e\cos\phi}{1-e^2} - 1\right)}$$ and

$$v_{Tot} = \left(1 + \frac{m_P}{m_S}\right)\sqrt{\left(\frac{2\pi GM}{P}\right)^{2/3}\left(\frac{2(1+e\cos\phi}{1-e^2} - 1\right)}.$$

2) $\sin\varphi$:

Considering the ellipse:



$$\sin\varphi = \sqrt{\frac{(r_P + s_P)^2 - d^2}{4 r_P s_P}}$$

$$\begin{cases} r_0 = r(\phi = 0) = \dfrac{a_P(1-e^2)}{1+e} \\ d = 2a_P - 2r_0 = 2\left(a_P - \dfrac{a_P(1-e^2)}{1+e}\right) = 2a_P\left(1 - \dfrac{(1-e^2)}{1+e}\right) \\ r_P = \dfrac{a_P(1-e^2)}{1+e\cos\phi} \\ s_P + r_P = const = s_0 + r_0 = 2a_P \Rightarrow s_P = 2a_P - r_P \end{cases}$$

Inserting these relations into the $\sin\varphi$ formula, we have:

$$\sin\varphi = \sqrt{\frac{1-e^2}{(1+e)^2}(1+e\cos\phi)\left(\frac{1}{2} - \frac{1}{1-e^2}\right)}$$

Finally, the central transit duration can be written:

$$D_{Max} = \frac{2R_{Tot}}{\left(\dfrac{2\pi GM}{P}\right)^{1/3}\left(\dfrac{2(1+e\cos\phi}{1-e^2} - 1\right)^{1/2}\left(\dfrac{1-e^2}{(1+e)^2}(1+e\cos\phi)\left(\dfrac{1}{2} - \dfrac{1}{1-e^2}\right)\right)^{1/2}\left(1 + \dfrac{m_P}{m_S}\right)}$$

We now define:

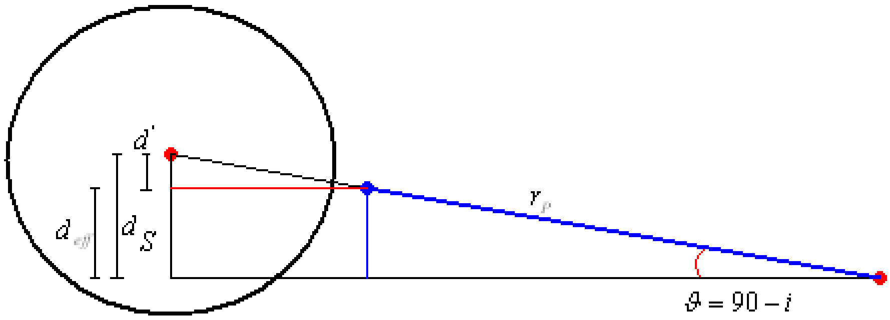
$$\left(1 + \frac{m_P}{m_S}\right) = \mathrm{A}$$

$$\left(\frac{1-e^2}{(1+e)^2}(1+e\cos\phi)\left(\frac{1}{2} - \frac{1}{1-e^2}\right)\right)^{1/2} = \mathrm{B}$$

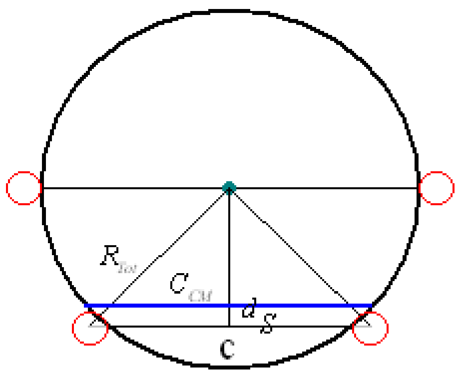
; A and B are correction values and account for the star orbital motion around the center of mass and for the geometry of the transit.

$$D_{Max} = \frac{2R_{Tot}}{\left(\frac{2\pi GM}{P}\right)^{1/3}\left(\frac{2(1+e\cos\phi}{1-e^2}-1\right)^{1/2}}\frac{1}{\text{AB}} = \frac{D_0}{\text{AB}}$$

Inclination



Since the star is generally not on the focus of the ellipse, to calculate the inclination, which is a function of the chord C, we must insert C_{CM} that is the calculated chord as if is the star is on the focus:



$$d_{eff} = r_P \cos i$$

$$C_{CM} = 2\sqrt{R_{Tot}^2 - d_{eff}^2} = 2\sqrt{R_{Tot}^2 - r_P^2\cos^2 i}$$

Considering the transit duration for i = 90° and that measured from the light curve, we have:

$$\begin{cases} D_1 = \dfrac{C}{v} \\ D_{Max} = \dfrac{2R_{Tot}}{v} \end{cases} \rightarrow \frac{D_{Max}}{D_1} = \frac{2R_{Tot}}{C} = \frac{R_{Tot}}{\sqrt{R_{Tot}^2 - r_P^2\cos^2 i}}$$

Resolving for i and eliminating r_P :

$$i = \arccos\left[\frac{R_{Tot}\sqrt{D_{Max}^2 - D_1^2}}{\left(\frac{GP^2M}{4\pi^2}\right)^{1/3}\left(\frac{1-e^2}{1+e\cos\phi}\right)D_{Max}}\right]$$

- **Radius:** $R_P = R_* \sqrt{\Delta F}$
- **Mass:** $M_P = \frac{M \sin i}{\sin i}$
- **Average density:** $< \rho >= \frac{M_P}{\frac{4}{3}\pi R_P^3}$